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Professional Advice: The Theory of Reputational Cheap Talk*

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Abstract

Professional experts offer advice with the objective of appearing well informed. Their ability is evaluated on the basis of the advice given and of the realized state of the world. This situation is modeled as a reputational cheap-talk game in which the expert receives a signal of continuously varying intensity with ability-dependent precision about a continuum of states. Despite allowing an arbitrarily rich message space, at most two messages are sent in equilibrium. The expert can only credibly transmit the direction but not the intensity of the information possessed. Equilibrium advice is then systematically less informative than under truthtelling.

Keywords: Reputation, cheap talk, advice, herding.

JEL Classification: D72 (Economic Models of Political Processes), D82 (Asymmetric and Private Information), D83 (Search, Learning, and Information), G20 (Financial Institutions and Services), J30 (Wages, Compensation, and Labor Costs).

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"Much has been written about the doubtful accuracy of economists' predictions. ... they are better at predicting the direction than the actual magnitude of events. ... This is disappointing, but it does not mean that economics is not a science." ('Economics', Encyclopaedia Britannica Online).

1. Introduction

Professional advisers are often concerned with their reputation rather than with the decisions made on the basis on their recommendations. Take for example business consultants' concern for the perceived quality of their services, managers' interest in promoting their careers, and politicians' pursuit of re-election. Though empirical studies have confirmed the importance of reputation in the financial industry, the theory of information transmission by professional advisers is still in its infancy.[1] This paper investigates from a theoretical point of view how the implicit incentives in the labor market and political system affect the information revealed by advisers concerned with their reputation.

We model strategic revelation of unverifiable information by a professional adviser seeking to develop a reputation for being well informed.[2] Advisers are assumed to have different degrees of expertise, i.e., informativeness of their signal structure. In our model the expert is assumed to receive a continuous signal of ability-dependent precision about the state of the world. The expert then reports to an evaluator, in a setting where no proof can be given to substantiate the recommendation. The state of the world is subsequently revealed to the evaluator, who combines it with the recommendation to update the belief regarding the expert's ability. This belief is referred to as reputation and determines the expert's future prospects and payoff.

As noted by Welch (2000), if analysts have a continuous message space it is in principle possible to invert each analyst's (supposedly separable) strategy thereby uncovering their private signals. If analysts release their reports sequentially and incorporate other analysts' information in their recommendations, the most recently issued report should efficiently aggregate the private information held by all analysts. For this reason, Welch studies analysts with an exogenously coarse message space (consisting of recommendations like "sell", "hold", and "buy") in order to obtain herding along the lines of Banerjee (1992)

[1] See the following recent empirical papers: Lamont (2002) on macroeconomic forecasters, Ehrbeck and Waldmann (1996) on three-month U.S. Treasury bills interest rate forecasters, Graham (1999), Hong, Kubik and Solomon (1999), Welch (2000), and Zitzewitz (2001a) on security analysts, and Chevalier and Ellison (1999) on mutual fund managers.

[2] See Sobel (1985), Benabou and Laroque (1992), and Morris (2001) for models of reputation building about preferences rather than quality of information possessed.

and Bikhchandani, Hirshleifer and Welch (1992). This paper shows that in reputational cheap talk equilibrium the message space is endogenously coarse.

Contrary to naive intuition, experts wishing to be perceived as accurate will not truthfully reveal their private information. Suppose that the evaluator presumes a fully separating strategy whereby the expert's signal can be inferred from the message reported. We prove that the signal bringing the highest reputational payoff is only rarely the one privately possessed by the expert. Generically the expert will wish to lie, claiming to possess the most advantageous signal. In the special case in which the signal alone conveys no information about the expert's ability, the expert has an incentive to bias the report towards the prior belief. Intuitively, the expert wants to give the impression of having a more informative signal than she does. Hence, truthtelling cannot be sustained in equilibrium. An expert who desires to impress a rational audience is unable to communicate all the information possessed. As a result professional advice cannot be taken at face value. Because part of the information possessed by experts cannot be credibly conveyed to the receivers, there is a welfare loss to society.

The model features cheap talk: the expert (sender) cares about the receiver's response (i.e., the evaluation of ability), but does not bear a direct cost from the message sent. Our finding that equilibrium communication by a professional adviser is necessarily coarse is reminiscent of Crawford and Sobel's (1982) result in the canonical model of partisan advice.[3] In our setting this result holds for "well behaved" information structures, as explained in Section 3. When senders with different information rank differently the receiver's evaluation of ability following the various messages sent, it is possible for some information to be communicated in equilibrium. For this to be the case, it is necessary that the evaluator receives ex post some additional information about the state.[4]

The endogenous coarseness of equilibrium communication is the starting point of our analysis. In order to characterize the structure of equilibria, we focus on the natural case of an expert who receives a signal of continuously varying intensity with ability-dependent precision about a continuum of states. With this special signal structure the most informative equilibrium is either binary or completely uninformative. In either case, a reported message pools many signals, and is therefore less precise than the sender's

[3]In the cheap-talk model of Crawford and Sobel (1982) a privately informed sender is interested in influencing the decision taken by a receiver. In contrast with the case of delegation considered by Holmström (1977), in cheap talk the receiver cannot commit to take any decision other than the ex-post optimal one given the information communicated by the sender. Crawford and Sobel find that some communication is possible when the sender and the decision maker have sufficiently congruent preferences.

[4]While in Crawford and Sobel's model the sender is always better informed than the receiver, in our setting the evaluator observes an additional signal (the realization of the state) before taking the action (evaluation of the sender).

true signal. Roughly speaking, the sender can at best communicate the direction of her information but cannot accurately convey its intensity. The report not only garbles the information about the state of the world, but also about the expert's true ability.

Bayarri and DeGroot (1988) and (1989) were the first to analyze an expert's incentive to manipulate the information reported in order to gain influence. They posited that the weight given to an expert is proportional to an expert's prior weight and the predictive density that the expert had assigned to the outcome that turns out to be actually observed.[5] In their setting, experts who maximize their own weight by optimally choosing the predictive distribution to report, typically do not want to honestly report their posterior belief. Our model departs from the Bayesian statistics literature in two important ways. Firstly, rather than assuming an ad hoc updating rule for the weights, we follow the lead of Holmström (1982/1999) by positing optimal updating on the quality of the expert's information.[6] The evaluator (or "market") is essentially a statistician and makes optimal use of all information available to form the posterior belief on the informativeness of the expert's signal. The second innovation with respect to the Bayesian statistics literature is that we not only characterize the incentives to deviate from honest reporting in our setting, but we also study the equilibrium of the game.

In their pioneering paper, Scharfstein and Stein (1990) analyzed the equilibrium of a two-period version of a reputational cheap talk model with a different sender in each period.[7] For simplicity they considered a model in which signals, states, and ability types are all binary. With two signals there is a perfectly informative equilibrium whenever an informative equilibrium exists (Ottaviani and Sørensen (2000)), so there is no manifestation of coarseness. While Scharfstein and Stein (1990) fixed the prior on the state such that there exists an informative equilibrium in the first period, in our continuous version of the static one-agent model we treat the prior on the state parametrically. In our more general formulation of the static model we can find conditions for coarseness and other qualitative properties of the equilibrium which could not be detected in their binary signal model.

The theory of reputational cheap talk can be applied to a number of social situations.

[5] This happens naturally if a linear opinion pool is used (e.g., see Genest and Zidek (1986)).

[6] Section III of Holmström (1982/1999) contains the first formulation of a reputational model where more able managers have access to a more precise signal about an investment opportunity. Holmström uses an example to illustrate that managers might refrain from investment in order to shield themselves from the risk associated with learning about ability that would otherwise result. For a general analysis of the moral hazard problem presented instead in the first part of Holmström's paper see Dewatripont, Jewitt and Tirole (1999).

[7] Departing from Holmström (1982/1999), they assumed that the state of the world is eventually realized regardless of the report. We also assume that the sender's report does not affect the state or what the receiver can observe about it.

Consider a politician who derives private benefits from being reappointed by an electorate assessing her competence. If those politicians who are considered to be better informed are more likely to be re-elected, they are subject to the same incentives as our professional advisers.[8] Likewise, the reputational objective is natural when modeling conversation among people who share preferences about alternatives or who have a negligible effect on the final decision to be taken.[9] In a companion paper, Ottaviani and Sørensen (2001b) apply the theory of reputational cheap talk to the problem of strategic forecasting and compare its predictions to those of alternative theories.

The paper is organized as follows: Section 2 sets up the model. Section 3 addresses whether the revelation of information can be truthful. Section 4 characterizes the optimal deviation from truthtelling. Section 5 analyzes the reputational cheap talk equilibrium, discusses some important implications for herding that can be obtained in dynamic extensions of the model, and derives some comparative statics predictions. Section 6 briefly discusses the empirical predictions of this model. Section 7 performs some robustness checks and contains extensions useful for applied and empirical research. In particular, we allow the expert to also have private information about her own type, to be directly concerned about the accuracy of the decision made, and to compete directly with other experts. Section 8 concludes with a summary of the contributions of the paper. All proofs are collected in the Appendix.

2. Model

An expert of ability (or *talent*) type $t \in T \subseteq \mathbb{R}$ privately receives an informative *signal* $s \in S$ on the *state* of the world $x \in X$ with conditional probability density function (p.d.f.) $f(s|x,t)$. Assume x and t are statistically independent, with common non-degenerate prior beliefs $q(x)$ on state and $p(t)$ on ability. In order to keep the expert's private information uni-dimensional, we assume until Section 7.3 that the sender does not know her own ability type t. After observation of the non-provable signal, the expert (or *sender*) decides which *message* $m \in M$ to send. The message space is arbitrarily rich. A strategy of the sender is

[8] See Biglaiser and Mezzetti (1997) for a characterization of the bias induced by re-election concerns on the decisions made by politicians in a model in which ability adds instead to the value of the project undertaken. Heidhues and Lagerlöf (2001) analyze political competition when the electorate rewards the politician who is most likely to have committed to the right decision.

[9] Before making a decision, individuals exchange information by speaking to one another. For example, committee members typically select the relevant alternatives via open discussion. Conversation often takes place among people who are interested in developing their reputation of being well informed. After all, those with better reputation are more likely to gain influence in future decisions. See Ottaviani and Sø rensen (2001a) for a model of political debate among heterogenous experts motivated by their reputation as good experts.

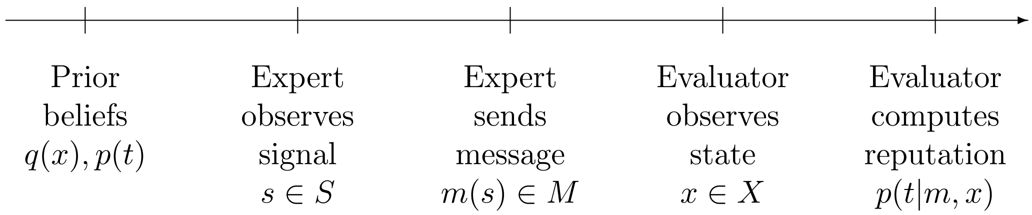

Figure 1: Time line for the model. The conditional density of the signal is $f(s|x,t)$. The expert's payoff is $\int_T v(t)p(t|m,x)\,dt$.

a mapping from signals into messages. The conditional probability that m is sent following signal s is denoted by $\mu(m|s)$.

The evaluator (or *receiver*) observes the message sent by the expert as well as the eventual realization of the state x. The evaluator's job is to compute the posterior reputation of the sender $p\,(t|m,x)$.[10] It is useful to think of the evaluator as being rewarded for predicting as accurately as possible the ability t of the expert based on the information (m,x) available. In order to calculate the posterior reputation, the evaluator must form a conjecture $\hat{\mu}$ on the strategy used by the sender. Given the conjecture, the evaluator computes the chances $\hat{f}\,(m|x,t) = \int_S \hat{\mu}(m|s)f(s|x,t)\,ds$ and $\hat{f}(m|x) = \int_T \hat{f}(m|x,t)p(t)\,dt$. The posterior reputation is then calculated by Bayes' rule, $p\,(t|m,x) = p\,(t)\,\hat{f}\,(m|x,t)\,/\,\hat{f}(m|x)$. The timing of the model is summarized in Figure 1.

The sender's preferences over posterior reputations are represented by the strictly increasing von Neumann-Morgenstern *utility function* $v(t)$.[11] The sender aims at maximizing $Ev(t)$, where the expectation is taken with respect to the posterior reputation $p(t|m,x)$.[12]

[10]Zitzewitz (2001b) proposes an alternative model in which the market evaluates the quality of the information contained in the forecast with a simple econometric technique, rather than via Bayesian updating.

[11]This is a psychological game in which the sender's payoff depends on the belief of the receiver. In line with Geanakoplos, Pearce and Stacchetti (1989), we assume that payoffs have an expected utility formulation. From a formal point of view, our formulation of an expert's preference for reputation is also similar to Bernheim's (1995) approach to measure people's preference for esteem. In his setting, esteem is equal to the expected value of a function over types evaluated using the posterior belief about an individual's type based on the information signaled in equilibrium. The function is non-monotonic because individuals desire to be perceived as having preferences close to a certain target level. In our setting the function is instead monotonic because the expert wishes to be perceived to have high ability.

[12]The payoff to the sender depends entirely on the receiver's belief and may be intangible. The payoff is tangible if it derives from the value of the services provided in a future second and last period by the expert, as in Holmström (1982). Truthful revelation is an equilibrium in this second period.

The reputational payoff of message m in state x is

$$W(m|x) \equiv \int_T v(t)p(t|m,x)\,dt, \tag{2.1}$$

so that the expected reputational payoff for a sender with signal s who sends message m is

$$V(m|s) = \int_X W(m|x)q(x|s)\,dx, \tag{2.2}$$

where the expert's posterior belief on the state x conditional on receiving signal s is given by Bayes' rule as $q\,(x|s) = f(s|x)q(x)/f\,(s)$, with $f\,(s|x) = \int_T f(s|x,t)p\,(t)\;dt$ and $f\,(s) = \int_X f(s|x)q\,(x)\;dx$.

Regardless of the privately observed signal, the expert wishes to induce the evaluator's most favorable beliefs. The preference ordering over reputation for expertise is therefore common across types. As first noticed by Seidmann (1990) in cheap-talk games with inter-type agreement, information can nevertheless be transmitted in equilibrium provided that the receiver's decision is based on some additional information. In our setting, the evaluator observes the state, not known to the sender when the message is sent. Messages sent correspond to lotteries over posterior reputations, which depend on the realization of the state. Depending on the evaluator's rule for calculating the posterior reputation, different messages may induce lotteries that are differently appealing to different types of experts.

3. Conditions for Truthtelling

By definition, truthful information transmission occurs when $M = S$ and the message sent equals the signal received, so that $\mu\,(s|s) = 1$. Assume for the moment that the receiver naively believes that the sender is applying this truthful strategy, so that $\hat{f}\,(m|x,t) = f\,(m|x,t)$. Is truthtelling then the optimal strategy for the sender? Whenever the answer is affirmative, truthtelling is a perfect Bayesian Nash equilibrium of the cheap-talk game.

We find below that truthtelling is an equilibrium in the standard model of the classical statistician performing a symmetric location experiment. But we quickly note that this finding is not robust in a Bayesian setting, as generic choices of the prior belief $q\,(x)$ and value function $v\,(t)$ render equilibrium truthtelling impossible.

3.1. Symmetric Location Experiments

We first show that truthtelling results in a completely symmetric location experiment with essentially no prior information. In order to have a proper uniform prior, the space

X should be compact – we will comment later on the important case where the prior is the improper uniform on the real line. Further, it would be inconvenient to make our symmetry assumptions below on a bounded subset of the real line. Assume then that the spaces X and S are both the unit circle, corresponding to the circumference of the unit ball in $\mathbb{R}^2$. A real number z indicates a point on the circle in the usual way, giving the anti-clockwise distance along the circumference from $(1, 0)$, the circle's origin in the plane.[13] To build a location experiment, let there be given p.d.f.s $g(s|t)$ over the unit circle, indexed by $t \in T \subseteq \mathbb{R}$, with these three properties:

(i) *Symmetry*: s is distributed on the circle symmetrically around 0, i.e. $g(s|t) = g(-s|t)$ for all $s \in [0, \pi]$.

(ii) *Unimodality*: s is distributed unimodally around 0, i.e. $g(s|t)$ is a decreasing function of $s \in [0, \pi]$.

(iii) *Monotone Likelihood Ratio Property* (MLRP): $g(s|t)/g(s|t')$ is strictly decreasing in $s \in [0, \pi]$ when $t' < t$, so that an s closer to 0 is better news for t.

The location experiment then has conditional p.d.f. given by $f(s|x, t) = g(s - x|t)$. It is simple to see that f inherits the symmetry and unimodality properties (around x) such that $f(x + s|x, t) = f(x - s|s, t)$ for all $s \in [0, \pi]$ and $f(x + s|x, t)$ is decreasing in $s \in [0, \pi]$. Clearly, $f(s|x) = g(s - x) = \int_T g(s - x|t)p(t)\, dt$ inherits these same properties for any prior $p(t)$.

Proposition 1 (Truthtelling in Location Experiment). *Consider a location experiment $f(s|x, t) = g(s - x|t)$ with g satisfying symmetry, unimodality and MLRP. If the prior $q(x)$ is the uniform distribution, there is a truthtelling equilibrium for arbitrary prior reputation $p(t)$ and increasing value function $v(t)$.*

This result crucially depends on the uniform prior on the state $q(x)$.[14] Truthfully reporting $m = s$ is then equivalent to reporting the mode of the symmetric posterior distribution $q(x|s)$. Since a signal s closer to the state x indicates a higher ability t by the MLRP and the state is concentrated around s, it is advantageous for the sender to send $m = s$ when the receiver interprets m as s. Truthtelling would instead be incompatible

[13] For instance, the numbers $-2\pi, 0, 2\pi$ all indicate the origin, while $\pi/2$ indicates the point $(0, 1)$ of the plane.

[14] Notice that Proposition 1 applies to more signal structures than those on the unit circle presented there. Assume that φ is a one-to-one mapping of $X = S$ into some other space $X' = S'$. Using φ we can transform $q(x)$ into a distribution on X', transform $g(s|t)$ into a distribution on S', and construct f as before. Then we find a new value function $V'(\varphi(m)|\varphi(s)) = V(m|s)$ and it is clear that the analysis carries over. For instance, with φ we could cut the circle open and straighten it out to an interval. The resulting family of signal distributions is no longer a proper location family, since it is wrapped at the ends of the interval, but it has $X, S \subseteq \mathbb{R}$.

with equilibrium for any location experiment with a proper prior belief on the state. To see this, consider the well-known normal location experiment with $s|x \sim N(x, 1/\tau)$ used by Ottaviani and Sørensen (2001b) to develop the theory of strategic behavior of professional forecasters. Unless the prior on the state is the improper uniform distribution on the real line, the report that guarantees the highest expected reputational payoff to the expert against the receiver's naive beliefs is not the best predictor of the state $E[x|s]$. For instance with normal prior on the state $x \sim N(\mu, 1/\nu)$, under some additional assumptions it can be shown that the best deviation is equal to $E[x|s' = E[x|s]]$, because a signal equal to the posterior mean is the one most likely to be observed by a well-informed expert. Since the posterior mean $E[x|s]$ is between the signal and the prior mean, an expert who is presumed honest by the market profitably deviates toward the prior mean.

To shed further light on Proposition 1, we now argue that truthtelling likewise results when a signal indicates that a state is infinitely more likely than all the other ones. This happens for instance when the state has an atomless distribution $q(x)$ and the signal a dichotomous distribution, whereby the expert receives perfect information ($s = x$) with probability t, and otherwise receives an uninformative draw from an atomless distribution $h(s)$. Formally, let $\delta_x(s)$ be the Dirac delta function, and assume that $X \subseteq S$. The signal is drawn from

$$f(s|x,t) = t\delta_x(s) + (1-t)h(s). \tag{3.1}$$

Receiving signal s, the posterior on the state has an atom at $x = s$ and a continuous density over all other x's. Moreover, the evaluator that receives $m = x$ concludes that the signal was derived from the perfectly informative distribution rather than the uninformative one, and that this is good news about the type. Conversely, $m \neq x$ is bad news. Thus, truthful reporting of the signal $m(s) = s$ constitutes an equilibrium, since any other signal would have probability zero to turn out to be correct. Formally:

Proposition 2 (Truthtelling in Dichotomous Experiment). *Truthtelling is an equilibrium in the dichotomous model with $h(s)$ and $q(x)$ atomless.*

3.2. Generic Impossibility of Truthtelling

We now show that truthtelling can only result in degenerate situations, as also independently observed by Campbell (1998) in a more special case. Assume that S, X are closed subsets of $\mathbb{R}$, and that S is convex (i.e., an interval). Assume that $f(s|x,t)$ is bounded and continuously differentiable in s, with f and f_s jointly continuous in (x,t).

We say that *local truthtelling is possible* at the signal $\hat{s} \in S$ if there exists an open interval $I \subset S$ containing $\hat{s}$, such that when the receiver anticipates truthtelling ($\hat{f}(m|x,t) =$

$f(m|x,t)$ for all $m \in I$) then $V(s|s) = \max_{m\in I} V(m|s)$ for all $s \in I$. In words, there is a whole interval around $\hat{s}$ where truthtelling by the sender is an optimal response to the receiver's anticipation of this. Local truthtelling immediately implies the first order condition

$$V_m(s|s) = 0 \tag{3.2}$$

for all $s \in I$. The thrust of our argument is to show that this identity cannot hold on an interval, unless if the model is degenerate.

A signal structure is defined to be *locally uninformative about talent* at $\hat{s} \in S$ if there exists an open interval $I \subset S$ containing $\hat{s}$ and functions $K(t)$ and $\kappa(s|x)$ such that $f(s|x,t) = K(t)\kappa(s|x)$ for all $s \in I$ and almost all $x \in X$ and almost all $t \in T$. This states that the conditional p.d.f. is separable in the (partly) observable outcome (s,x) and the unobservable talent t about which inference is made. The condition implies that the evaluator cannot use the pair (s,x) to make any discriminatory inference on t. Namely, for any two pairs (s,x) and (s',x') we find $p(t|s,x) = p(t|s',x')$.[15]

Local truthtelling is possible at $\hat{s}$ when there is local uninformativeness at $\hat{s}$, since the posterior reputation is entirely independent of the message sent. This complete indifference trivially results in truthtelling. The crux of the model is that the sender affects the posterior reputation $p(t|m,x)$ through the message m depending on the realization of the state x. The assumption of global uninformativeness needed to obtain global truthtelling is then unduly restrictive. Unless the signal is globally uninformative, there is scope for strategic manipulation of the posterior belief.

Theorem 1 (No Truthtelling). *Assume that S, X are closed subsets of $\mathbb{R}$, and that S is convex (i.e. an interval). Assume that $f(s|x,t)$ is bounded and continuously differentiable in s, with f and f_s jointly continuous in (x,t). If the signal structure is not locally uninformative about talent at $\hat{s} \in S$, local truthtelling at $\hat{s}$ is impossible for an open and dense set of prior beliefs $q(x)$ and value functions $v(t)$.*

If the signal is not locally uninformative, different message and state pairs (m,x) imply different posterior reputations. The sender is uncertain about the location of x, and through perturbations in the prior $q(x)$ we perturb the lotteries over reputations resulting from the available messages. Perturbations in $v(t)$ guarantee that this translates into a relative differentiation of the expected reputational value.

[15] Here the posterior reputation was stated under the assumption of truthful reporting. But it is easily verified that it holds for more general strategies. For any two messages m and m' which are sent only for signals in the interval I, and for any two states x and x', we have $p(t|m,x) = p(t|m',x)$.

The truthtelling condition (3.2) also suggests how to use explicit monetary incentives to reinstate truthtelling. If the message sent were verifiable and explicit incentives were allowed, truthtelling could be obtained by offering the reward schedule $R(m) = \int_{-\infty}^{m} V_m(\tilde{m}|\tilde{m})\, d\tilde{m}$ to the expert. Correspondingly, the ex-ante cost of implementing truthtelling would be $\int_S R(s) f(s)\, ds$. Notice that the cost could be lower if the reward were allowed to depend also on the realization of the state. For the rest of this paper we exclude the possibility of monetary incentives.

4. Optimal Deviation in Linear Model

For the remainder of this paper we posit that the distribution of the signal conditional on the state x and ability t is *linear* in $t \in [0,1]$,

$$f(s|x,t) = tg(s|x) + (1-t)h(s), \tag{4.1}$$

being a mixture between an informative and an uninformative experiment.[16] Better experts are more likely to receive a signal drawn from the informative $g(s|x)$ rather than the uninformative $h(s)$. In fact, a more talented expert receives better information in the sense of Blackwell. To see this, consider the garbling of s into $\widetilde{s}$ whereby $\widetilde{s} = s$ with probability $\tau < 1$, and otherwise $\widetilde{s}$ is independently redrawn from $h(s)$. Then

$$\widetilde{f}(\widetilde{s}|x,t) = \tau f(\widetilde{s}|x,t) + (1-\tau)h(\widetilde{s}) = f(\widetilde{s}|x,\tau t),$$

so that the garbled signal to an expert of ability $t > 0$ is distributed as the ungarbled signal to an expert of ability $\tau t < t$.

The linearity of $f(s|x,t)$ in t greatly simplifies considerations involving the expert's payoff. A strategy of the sender is a mapping from signals to messages, with $\mu(m|s)$ denoting the conditional chance that m is sent when s is the signal. When the receiver conjectures the strategy $\hat{\mu}$, he can compute $\hat{g}(m|x) = \int_S \hat{\mu}(m|s)g(s|x)\, ds$ and $\hat{h}(m) = \int_S \hat{\mu}(m|s)h(s)\, ds$. Then $\hat{f}(m|x,t) = t\hat{g}(m|x) + (1-t)\hat{h}(m)$ and $\hat{f}(m|x) = \int_T \hat{f}(m|x,t)p(t)\, dt$. Bayesian updating gives the posterior belief on ability $p(t|m,x) = \hat{f}(m|x,t)p(t)/\hat{f}(m|x)$. Substitution

[16] This linear model is well suited to study problems in information economics. While the similarity with Hart and Holmström's (1987) linear distribution function condition is only superficial, the connection with Green and Stokey's (1980) success-enhancing model is deep rooted. In the success-enhancing model the experiment fails with positive probability, in which case the signal is uninformative about the state. Similarly, in the linear model, the signal comes from an uninformative experiment with positive probability. The main difference is that in the success-enhancing model the experimenter observes whether the experiment failed or not, while in the linear model the experimenter only knows the probability that the experiment is contaminated.

in (2.2) gives

$$
\begin{aligned}
V(m|s) &= \int_X \int_T v(t) \frac{t\hat{g}(m|x) + (1-t)\hat{h}(m)}{E[t]\hat{g}(m|x) + (1-E[t])\hat{h}(m)} p(t)\, dt\, q(x|s)\, dx \\
&= E[v(t)] + (E[tv(t)] - E[t]E[v(t)]) \int_X \frac{\hat{g}(m|x) - \hat{h}(m)}{\hat{f}(m|x)} q(x|s)\, dx,
\end{aligned}
$$

which depends on $p(t)$ only through Et. Notice that $E[tv(t)] - E[t]\, E[v(t)] > 0$ when v is strictly increasing and t does not have a degenerate prior distribution. In the linear model the expert's behavior is therefore independent of properties of the value function $v(t)$ other than that it is strictly increasing. The reason is that in model (4.1), posterior reputations $p(t|x,m)$ are unambiguously ranked (in the first-order stochastic sense) depending on the pair (x,m).

Lemma 1. *It is without loss of generality to let the expert have payoff*

$$V(m|s) = \int_X \frac{\hat{g}(m|x) - \hat{h}(m)}{\hat{f}(m|x)} q(x|s)\, dx, \tag{4.2}$$

a positive affine transformation of the original (2.2).

With $\hat{f}(m|x,t) = t\hat{g}(m|x) + (1-t)\hat{h}(m)$, the higher $\hat{g}(m|x)$ is relatively to $\hat{h}(m)$, the higher is the expert's reputation, for this corresponds to higher weight on the t term and lower weight on the $1-t$ term. The result that $W(m|x) = \left(\hat{g}(m|x) - \hat{h}(m)\right)/\hat{f}(m|x)$ clearly reflects this.

In the following we assume that S, X are subsets of $\mathbb{R}$. Assume that both $g(s|x)$ and $h(s)$ are twice continuously differentiable and that g satisfies the MLRP in (s,x). A necessary and sufficient condition for a linear signal structure to satisfy the MLRP in (s,x) for all t is $g_{sx}h > g_x h_s$. This follows from the observation $f_{sx}f - f_s f_x = t^2(g_{sx}g - g_s g_x) + t(1-t)(g_{sx}h - g_x h_s)$. This MLRP assumption, satisfied by model (5.1) below, is maintained throughout the paper.

A particular signal realization $\tilde{s}$ is *neutral* about the state, if $g(\tilde{s}|x)$ is constant in x. An expert who receives a neutral signal has posterior beliefs $q(x|\tilde{s}) = q(x)$: the signal is not informative about x since $f(\tilde{s}|x,t)$ is independent of x.

Let us now revisit the impossibility of truthtelling within the linear model. In response to naive beliefs, the ideal signal an expert wishes to send is different from the one observed. With a few restrictions on the model, we can predict that the direction of the deviation is towards the neutral signal:[17]

[17] For a similar but independently derived result see Campbell's (1998) Proposition 3.1.

Proposition 3 (Best Deviation). *Assume $g_{sx} > 0$ and that signal $\tilde{s}$ is neutral. Assume that any signal is uninformative about ability ($p(t|s) = p(t)$ for all s). The best deviation against naive beliefs is to report a signal s' strictly in between the neutral signal $\tilde{s}$ and the signal actually possessed s.*

This result requires stronger assumptions on the signal structure than Theorem 1, but is valid for *all* increasing value functions v. Its logic relies on the following three observations. First, higher realizations of the state x are better news about ability when signal s' such that $s' > \tilde{s}$ is understood to have been reported. Second, the sender with s such that $s > s'$ believes more in higher realizations of x the sender with s'. Third, the sender who reports truthfully is expecting the same value Ev regardless of the signal actually observed. Therefore, the sender with s has a higher expected reputational payoff from reporting $s' \in (\tilde{s}, s)$ compared to that of the sender with signal s', itself equal to the truthtelling value Ev.[18]

5. Equilibrium in Multiplicative Linear Model

In order to characterize the equilibrium we impose from now on further restrictions on the signal distribution. We adopt the *multiplicative linear model*: the signal conditionally on the state $x \in X = [-1, 1]$ and ability type $t \in T = [0, 1]$ is distributed according to the density

$$f(s|x,t) = t\frac{1+sx}{2} + (1-t)\,\frac{1}{2} = \frac{1}{2}(1+stx), \tag{5.1}$$

with $s \in S = [-1, 1]$, as illustrated in Figure 2.

This signal structure satisfies the monotone likelihood ratio property (MLRP) in s, x for any value of $t > 0$: The likelihood ratio $f(s|x,t)/f(s|x',t)$ is increasing in s for $x > x'$. Clearly, also $f(s|x) = \int_T f(s|x,t)p\,(t)\;dt = f(s|x, Et)$ satisfies the MLRP. Notice also that this signal structure is boundedly informative about the state (the only exception to this being for $t = 1$, when the most extreme signals $s = \pm 1$ rule out the most extreme states $x =$

[18] Although driven by different forces, this result is reminiscent of the "yes-men" effect analyzed by Prendergast (1993). He considers an agent who has access to two private signals, one on the state of the world and the other on the principal's private signal on the state. In order to induce the agent to gather and report information, the principal commits to a reward scheme based on the difference between the agent's report and the principal's signal. This commitment results in the agent honestly reporting her best estimate of the principal's private signal. But since the agent's report contains information from her two sources, the principal can extract only imperfectly the agent's direct signal about the state. While in Prendergast's model the agent does not sufficiently move away from the principal's opinion, in our model the agent does not move away from the neutral signal. In both models, information transmission is therefore inefficient. (Ewerhart and Schmitz (2000) have shown that if instead the agent in Prendergast's model is also asked to report her private information, efficiency is restored).

∓ 1 respectively). This model satisfies two other important properties: First, signal $s = 0$ is neutral with respect to both state ($q(x|s=0) = q(x)$) and ability ($p(t|s=0) = p(t)$). Second, when the prior on the state has zero mean $Ex = 0$, any signal is uninformative about ability, i.e. $p(t|s) = p(t)$.[19]

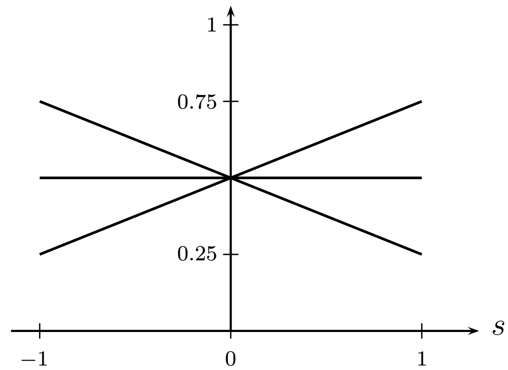

Figure 2: Graphs of the conditional densities $f(s|x,t) = (1 + stx)/2$ for fixed $t = 1/2$ and three values of $x = -1, 0, 1$. The downward sloping line corresponds to the case with $x = -1$, the flat one to $x = 0$, and the increasing one to $x = 1$. Intermediate values of x would give intermediate lines. Each line (other than the one corresponding to $x = 0$) becomes steeper as t increases.

Notice that the widely used symmetric binary model has the same generalized p.d.f. (5.1), with $S = X = \{-1, 1\}$ and $T = \{\underline{t}, \bar{t}\}$, where $0 \leq \underline{t} < \bar{t} \leq 1$ (see e.g. Scharfstein and Stein (1990)). It is useful to think of a signal satisfying (5.1) as being binary, but of a continuously varying intensity level. The multiplicative linear model is then a natural generalization of the binary-signal model to allow for a continuum of states, signals, and ability types in a tractable way and might be useful in other problems in information economics.[20] By re-normalizing the support of S and X to the unit interval, it is immediately seen that this is the Farlie-Gumbel-Morgenstern distribution with uniform marginals (cf. Conway (1983)).

With this additional restriction we can derive strong characterization results. There can be only partition equilibria with endogenously coarse communication (Section 5.1), the only informative equilibria are binary and there is no informative equilibrium at all when

[19]This assumption was made by Scharfstein and Stein (1990) and is maintained in Campbell (1998). In this case the sender does not learn anything about own ability by observing the signal, so that the message sent cannot signal any such knowledge. This assumption amounts to a degenerate restriction on the set of prior beliefs on the state.

[20]Special versions of this model have been extensively used in economics. See e.g. Lohmann's (1994) generalization of the binary model and Piccione and Tan's (1996, page 504) example of a signal structure with an uninformative signal.

the prior belief on the state is sufficiently concentrated (Section 5.2). After presenting an extended example with binary state (Section 5.3), we briefly discuss issues arising when extending the model to allow for sequential advice by different experts (Section 5.4) and derive some comparative statics results (Section 5.5).

5.1. Interval Equilibria

The multiplicative linear model (5.1) satisfies the conditions of Theorem 1 for generic impossibility of truthtelling and Proposition 3 for the best deviation provided that $Ex = 0$. In the multiplicative linear model we have $\hat{f}(m|x,t) = (1 + E[s|m]tx)\,\hat{\mu}(m)/2$, where $\hat{\mu}(m) = \int_S \hat{\mu}(m|s)ds$, and $E[s|m] = \int_S s\hat{\mu}(m|s)\,ds/\hat{\mu}(m)$, so that

$$W(m|x) = \frac{\hat{g}(m|x) - \hat{h}(m)}{\hat{f}(m|x)} = \frac{E[s|m]x}{1 + E[s|m]xEt}. \tag{5.2}$$

We can then derive the following stronger result:

Proposition 4 (Absolutely No Truthtelling). *Local truthtelling at any $s \in [-1,1]$ is impossible for all non-degenerate priors $q\,(x)\,, p\,(t)$ and strictly increasing value functions $v\,(t)$.*

Having ruled out perfectly separating equilibria, we now show that equilibria have a partition structure whereby connected sets of signals are pooled. Notice that the following sorting condition holds:

$$\frac{\partial^2 W(m|x)}{\partial E[s|m]\partial x} = \frac{1 - E[s|m]xEt}{(1 + E[s|m]xEt)^2} > 0. \tag{5.3}$$

Messages corresponding to signals with higher mean give higher payoff the higher the state of the world.

Consider two possible messages, m and m' where m' is higher than m in the sense that $E[s|m'] > E[s|m]$. Then (5.3) implies that the higher message yields a payoff increasing in x. Since experts with higher signals believe in higher states, we can establish the following monotonicity property:

Proposition 5 (Monotonicity). *$V(m'|s) - V(m|s)$ increases in s if $E[s|m'] > E[s|m]$.*

Incentive compatibility implies that if two messages have $E[s|m'] > E[s|m]$, all expert types sending message m' have higher signals than those sending m. This implies that each message m sent in equilibrium corresponds to signals that belong to some interval subset of S. By Proposition 1 we know that there cannot be truthful reporting in any subinterval of S. Hence, the typical message interval has a non-empty interior, although there may be occasional isolated one-point intervals:

Proposition 6 (Partitions). *All perfect Bayesian equilibria have interval messages.*[21]

As is typical in cheap-talk games, a completely uninformative (pooling or babbling) equilibrium always exists. If the evaluator expects all messages to be uninformative, the senders have no choice but to pool on the relevant messages. Rather than discussing equilibrium selection, we characterize the set of all perfect Bayesian equilibria.

5.2. Binary Equilibria

The partition structure of the equilibria in our professional model is similar to one found by Crawford and Sobel in the partisan setting, but it is driven by different forces. Differently from their setting, there is no natural notion of closeness between a professional adviser's objective and the receiver's evaluation objective. We find in the multiplicative linear model all equilibria are binary: only two messages are sent, one for $s \geq a$ and the other for $s < a$, where $a \in (-1, 1)$. The proof proceeds by contradiction. Suppose that more than two messages were sent in equilibrium. By continuity, an expert with a signal at the border between two adjacent messages must be indifferent between them. The contradiction follows from the fact that the two indifference conditions at the extremes of an intermediate message are incompatible.

Proposition 7 (Partition Size). *In the multiplicative linear model all informative equilibria are binary.*

This result is striking, but it is quite special to the multiplicative linear model (5.1).[22] We now provide a simple example of an equilibrium with more than two messages in a statistical model belonging to the linear class (4.1). Let $X = \{-1, 1\}$, $S = [-1, 1]$, $g(s|x) = (1 + sx)/2$, and $h(s) = \gamma + \delta s^2$. Clearly, $\gamma = 1/2 - \delta/3$ in order for $h(s)$ to be a density. Furthermore, $\gamma > \delta$ for the MLRP to be satisfied, so that we need $\delta < 3/8$. Set for example $\gamma = 5/12$, $\delta = 1/4$, $Et = 1/2$, and $\Pr(x = 1) = \Pr(x = -1) = 1/2$, and look for a symmetric equilibrium with three messages. It is easy to check numerically that the three messages $\{[-1, -a], [-a, a], [a, 1]\}$ with $a \approx .80218$ constitute an equilibrium.

[21] It is technically possible to construct non-interval equilibria where two messages m and m' sent in equilibrium have $E[s|m] = E[s|m']$. In that case the two corresponding messages, m and m' convey equal information about x, t. Indeed, $f(m|t, x) = f(m'|t, x) = (1 + E[s|m]tx)/2$ for all t, x. Thus the two messages might as well be pooled into one message, and we restrict attention to the interval equilibria.

[22] In the context of the normal learning model, Ottaviani and Sørensen (2001b) have shown that there is always a binary equilibrium, but have not been able to prove that there are no equilibria with more than two partitions. This is because the cumulative distribution of the normal distribution is not analytically tractable. This problem is clearly overcome by the multiplicative linear model introduced in this paper.

Notice that this model does not have the uninformativeness property, i.e. it is not possible to have a prior on the state such that any signal is uninformative about ability.

Returning to our multiplicative linear model (5.1), we now characterize the binary equilibria. Let m denote the message sent for $s \in [-1, a]$ and m' the message sent for $s \in (a, 1]$, with $-1 < a < 1$. The indifference condition $V(m|a) = V(m'|a)$ is

$$\int_X \frac{x\,(1 + axEt)}{[2 + (a-1)xEt]\,[2 + (a+1)xEt]} q(x)\,dx = 0. \tag{5.4}$$

If and only if $a \in (-1, 1)$ solves this equation, messages $[-1, a]$ and $(a, 1]$ constitute a binary equilibrium.

We now identify a prominent instance in which a binary equilibrium exists. We ask, when is there a symmetric equilibrium with $a = 0$? Inserting in (5.4) we find

$$\int_X \frac{x}{(2 - xEt)\,(2 + xEt)} q(x)\,dx = 0.$$

As the function $x/\left[4 - x^2(Et)^2\right]$ is anti-symmetric around $x = 0$, we conclude that the symmetric equilibrium exists when the distribution of x is symmetric around 0.

Proposition 8 (Existence of Binary Equilibrium). *When the prior on x is symmetric around 0, there is a symmetric binary equilibrium with messages $[-1, 0]$ and $(0, 1]$.*

The intuition is straightforward. The sender learns nothing about the state when receiving the neutral signal $s = 0$, so that with the symmetric prior (and posterior) on the state it looks equally attractive to send either the high or the low message.

Notice that at the ends of the interval, $a = -1$ and $a = 1$, the left-hand side of (5.4) is equal to $Ex/4$. The integral varies continuously with a, showing that the number of binary equilibria must be even. Thus, when informative equilibria exist, generically in the prior on state, there are multiple such equilibria. To better understand this multiplicity, consider the equilibrium condition, $V((a, 1]|a) - V([-1, a]|a) = 0$. On the one hand, $V((a, 1]|s) - V([-1, a]|s)$ is increasing in s for any a, i.e. holding fixed the receiver's beliefs the sender with higher signal likes better the high message (Proposition 5). On the other hand, $V((a, 1]|s) - V([-1, a]|s)$ is decreasing in a for any s, i.e. holding fixed the sender's signal the higher message becomes less appealing when the receiver's beliefs move up. The balance between these two opposed effects determines whether $V([a, 1]|a) - V([-1, a]|a)$ is increasing or decreasing in a. Multiplicity of equilibria results from the fact that $V([a, 1]|a) - V([-1, a]|a)$ is equal to 0 for several values of a.

Finally, assume that the prior is highly concentrated near some $x > 0$. Any signal s is of bounded informativeness about states of the world, so the posterior $q(x|s)$ is still concentrated around x. Whenever the state turns out positive, it is favorable to the expert's reputation to report a message with $E[s|m] > 0$. If it were possible to send a message with $E[s|m] > 0$, the expert would want to send this message, regardless of the signal s actually received. This cannot hold in equilibrium, since $E[s|m] = 0$ when all signals are pooled into one message:

Proposition 9 (No Informative Equilibrium). *If the prior distribution on the state is concentrated sufficiently close to any $x \neq 0$, there exists no informative equilibrium.*

Intuitively, when the prior is concentrated enough there cannot be any informative equilibrium, because all experts wish to bias their signal in one direction.[23] Note that this result does not hold for $x = 0$, since Proposition 8 guarantees the existence of an informative equilibrium for any (symmetric) prior arbitrarily concentrated on 0. Yet, even in that case the messages convey very little information about ability.

The most informative equilibrium is either binary or even completely uninformative. Since reported messages pool many signals, they are far less precise than the sender's true signal. The sender can communicate at most the direction of her information but cannot convey its intensity. There is pooling on the intensity dimension, since experts would always want to pretend to have more precise information. Rationality of the evaluator makes this incentive self defeating.

5.3. Binary State Example

We now offer a pictorial depiction of the equilibria in a simple example. Assume that the prior distribution of x is concentrated on -1 and $+1$, with q being the prior probability of state $+1$. Now (5.4) can be re-written as

$$\frac{q}{1-q} = \frac{1-aEt}{1+aEt}\frac{2+(a-1)\,Et}{2-(a-1)\,Et}\frac{2+(a+1)\,Et}{2-(a+1)\,Et}, \tag{5.5}$$

a third-order polynomial equation in a. Denote the right hand side of (5.5) by $\rho(a, Et)$, plotted in Figure 3 for $Et = 1/2$. For $q = 1/2$ the equation has one solution in $(-1, 1)$, so that $a = 0$ is an equilibrium. The equation has two solutions in $(-1, 1)$ when $q \in (1-\bar{q}, \bar{q})$ with $\bar{q} > 1/2$, one solution when q is equal to $1-\bar{q}$ and $\bar{q}$, and no solution in $(-1, 1)$ whenever $q > \bar{q}$ and $q < 1-\bar{q}$. As illustrated in the figure, for the special case $Et = 1/2$ we have $\bar{q} = \frac{1}{2} + \left(\frac{4-\sqrt{11}}{60}\right)\sqrt{\left(7-2\sqrt{11}\right)} = .5069$. It is easy to show that there can be no solution to (5.5) for $q < 1/3$ or $q > 2/3$, no matter how good is the prior reputation Et. No expert can speak credibly for these prior beliefs.

[23] As shown in Ottaviani and Sørensen (2001b), the result does not hold in the normal learning model, in which signals are unboundedly informative about the state.

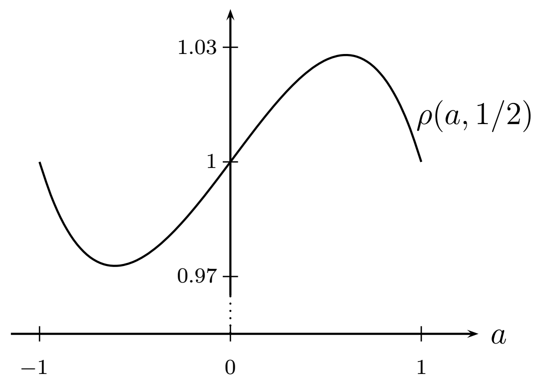

Figure 3: Graph of the right hand side of (5.5) when $Et = 1/2$.

5.4. Dynamic Extensions and Herding

Proposition 9 is an important building block in a model of herding. Experts who give advice in sequence, learn about the state of the world by listening to each others' recommendations. As more experts speak informatively, by the law of large numbers the beliefs of later experts become ever more concentrated on the true state x. According to Proposition 9, if the belief becomes sufficiently concentrated in finite time, experts cannot be informative any longer and learning stops. This is the logic of statistical herding (Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992)) applied to this reputational model.[24] As shown by Smith and Sørensen (2000), with continuous signals the belief may not reach the herding limit in finite time – still, learning grinds to a halt.

As an illustration of the working of the dynamic model, consider the case where two experts ($i = A$ and B) decide in sequence. Each manager i receives a signal s^i, independent conditionally on the state x and distributed according to (5.1) with binary state $X = \{-1, 1\}$. The initial prior on the state is $q^A = \Pr(x = 1) = 1/2$, so that the prior expectation on state is $Ex = 0$ and the prior expectation on abilities are $Et^A = Et^B = 1/2$. It follows from the analysis of the one-period problem that in the (unique most-informative equilibrium) the first agent A reports the high message $m^A = [0, 1]$ when observing $s \in [0, 1]$ and the low message $m^A = [-1, 0]$ when observing $s \in [-1, 0]$. With conditional independent signals, the equilibrium for the second agent B depends exclusively on the posterior belief on the state after observation of agent A's behavior. For example, B's prior belief on the state upon observation of message $[0, 1]$ sent by A is

$$q^B = \Pr\left(x = 1 | m^A = [0,1]\right) = \frac{\Pr\left(m^A = [0,1] | x = 1\right) \Pr(x = 1)}{\Pr\left(m^A = [0,1]\right)} = \frac{1}{2} + \frac{Et^B}{4} = .625.$$

[24]See Ottaviani and Sørensen (2000) on the connection between statistical with reputational herding models.

The equilibrium for the second agent is then determined by Figure 3. The second agent herds since $q^B = .625 > \bar{q} = .5069$ for $Et^B = 1/2$. When agent A sends a high message, so does agent B. Similarly, when A sends the low message, B also sends a low message. We can conclude that the fact that differential conditional correlation is necessary to obtain herding is not a robust finding of the binary signal model, even when Scharfstein and Stein's (2000) strong definition is used: here "B *always* ignores his own information and follows agent A" under conditional independence.[25]

5.5. Comparative Statics

We are now ready to address some natural comparative statics questions. Notice that a signal is more informative about ability the larger is $|x|$. Similarly, a message – which consists of a garbling of the signal – is more informative about ability the larger is $|x|$. Since the equilibrium strategy may be asymmetric (when s is garbled into m) nothing can be said a priori on how informative is m about t.

Do Better Reputed Experts Send More Informative Messages? By construction of the model, ex ante better experts are Blackwell-more informed about x. We now show by way of example that better experts' messages need not be Blackwell-more informative because of the equilibrium garbling of the signal. Take $q = .505$ in the binary-state illustration above and consider two experts, the first with $Et = .49$ and the second with $Et = .5$. Assume that the informative equilibrium with a threshold nearest to 0 has been selected – a similar example proves our point for the other informative equilibrium. When message $s \in [-1, a]$ is observed, the posterior belief is

$$q(x = 1|m = [-1, a]) = \frac{[2 + (a-1)Et]\, q}{[2 + (a-1)Et]\, q + [2 - (a-1)Et]\,(1-q)}$$

A similar expression defines $q(x = 1|m = (a, 1])$. The following is based on numerical solution of equation (5.5). The expert with $Et = .5$ has an equilibrium with $a = .329$ yielding $q(x = 1|m = [-1, a]) = .42101$ and $q(x = 1|m = (a, 1]) = .67058$. The expert with $Et = .49$ has an equilibrium with $a = .357$ resulting in $q(x = 1|m = [-1, a]) = .42617$ and $q(x = 1|m = (a, 1]) = .67072$. In a decision problem with two actions and indifference at a belief in the interval $(.67058, .67072)$, the expert with $Et = .5$ is of no value while the expert with $Et = .49$ transmits valuable information.

[25] Note that this conclusion is valid regardless of whether Scharfstein and Stein's (1990) non-informativeness condition is satisfied or not. The non-informativeness condition is satisfied in this example, but would fail for a slightly different prior on the state.

Are Better Reputed Experts Credible for a Larger Set of Priors? Consider an expert B with a prior reputation $p(t)$ which is FSD-better than the prior reputation of expert A. Is B non-herding for a wider set of priors on the state of the world? Above we have seen that the issue of credibility depends on the prior reputation only through Et, since it is a question of solving (5.4) with $a \in (-1, 1)$. We cannot answer this question in the general case, but the answer is affirmative for the binary-state of the world case. To see this, note that for any $a \in (0, 1)$, the right hand side of (5.5) is larger the larger is Et.

How Does the Equilibrium Change with the Prior Beliefs? Departing from the symmetric case, we can analyze the direction of change for a away from 0 when we skew the state distribution in the multiplicative linear model. A first-order stochastic dominance (FSD) increase of x makes the original high message m' more attractive than the low message. In the new equilibrium the threshold of indifference between the two messages must therefore change. If the threshold were to decrease, an even larger set of experts would wish to send the high message m', so that the indifference threshold would move further down. This would make m' even more attractive in an unstable process which does not lead to a new equilibrium. In order to re-equilibrate the attractiveness of the two messages, the threshold must instead move up:

Proposition 10 (Comparative Statics). *Departing from a prior $q(x)$ symmetric around $x = 0$, a first-order stochastic dominance increase in $q(x)$ results in a new binary equilibrium with a higher threshold of indifference.*

6. Predictions

The equilibrium loss of information typically results in a welfare loss for the decision maker. Likewise, future employers of the expert are interested in learning as much as possible the expert's true ability, and so they would prefer that the signal were not garbled. If the value function $v(t)$ is linear, the sender's ex ante expected reputational value of sending any message profile is equal to its prior value. Therefore, the expert is indifferent in ex-ante terms between the different equilibria. In expectation, no one benefits from the fact that information transmitted in equilibrium is less precise than the information possessed by the expert.

For the application of this model to the predictions of professional experts, we need to discuss how information is communicated. In equilibrium the receiver understands that signals in a certain interval ($s \in m$) are pooled into the same message by the sender. But the actual cheap-talk language is using arbitrary messages m which need not live in the same space as the signals. Given this arbitrariness, an empirical comparison of the experts' literal statements with the outcome of the predicted variable x is difficult.

The advice given by the expert is typically used by a decision maker, whose decision can only rely on the information about the payoff-relevant state x contained in the message. The decision maker's beliefs $f(x|m)$ as well as the decision taken on the basis of such belief are unambiguously determined in equilibrium. The natural language of forecasters dictates them to communicate this belief (or its mean $E[x|m]$) or to recommend the corresponding course of action. Alternatively, under delegation the action taken serves as the message. Statements in such languages can be easily compared with the realized state.

The resulting belief $f(x|m)$ is unbiased, being derived from Bayesian updating, but it is less accurate than the forecaster's private belief $f(x|s)$. If many identical experts with absolute reputational concerns are polled simultaneously, their forecasts should be very similar, concentrated on at most two different positions. Moreover, if their forecasts were replicated using their models of the economy, the empirically observed forecasts would appear very inaccurate, since the replication gives the more informative $f(x|s)$. The empirical forecast errors would likewise appear excessively correlated.

A direct test of our theory would be based on the regression

$$x = \alpha_0 + \alpha_1 m + \alpha_2 y + \alpha_3 s + \varepsilon, \tag{6.1}$$

where x is the realized state, m the forecast, y any publicly known variable at forecast time, s the private information of the expert, and ε the forecast error. *Unbiasedness* requires that, when y and s are excluded, the remaining coefficients are restricted to $\alpha_0 = 0$ and $\alpha_1 = 1$. *Efficiency* requires that all information available to the forecaster has no additional predictive power in the regression, i.e. $\alpha_2 = \alpha_3 = 0$. Identifying m with the prediction on the state $E[x|m, y]$, our reputational cheap talk model predicts unbiasedness and efficiency only with respect to public information $\alpha_2 = 0$. According to our coarseness result, the message sent is not a sufficient statistic for the expert's private information. Furthermore, it is easy to show that the MLRP of s, x implies the MLRP of s, x conditional on any realization m, when s is a Blackwell sufficient experiment for m (cf. Ottaviani and Prat (2001)). Thus our model predicts that $\alpha_3 > 0$. Direct test of this prediction would require access to the forecaster's private information, but this is rarely available. Rather than providing direct tests of reputational cheap talk, most of the existing empirical literature provides indirect evidence based on extensions of the basic model. These extensions are investigated in Section 7.[26]

[26]For further discussion of the predictions of the theory when applied to strategic forecasting we refer

7. Extensions

In order to derive additional testable predictions, the model is extended to account for some realistic features of advice. In Section 7.1 we show that communication is coarse also when the expert has a partial direct concern for forecast accuracy. In Section 7.2 we show that if instead the evaluator does not observe the state, an expert exclusively concerned about reputation cannot communicate any information. Section 7.3 shows that when the expert knows privately her own ability type, it is always possible to sustain a binary informative equilibrium. In Section 7.4 we investigate whether competition resulting from relative reputational concerns among advisors can improve communication.

7.1. Mixed Reputational and Statistical Objective

Forecasters are typically motivated at least in part by the accuracy of their forecasts. Similarly, professionals care not only about their reputation but also about the return of the decision made on the basis of their advice. The mixed objective model presented here allows us to investigate whether the coarseness result is robust to the introduction of a statistical component in the objective function.

In the mixed (M) model,

$$V^M(m|s) = \beta V(m|s) - (1-\beta)\int_X (E[x|m]-x)^2\, q(x|s)\, dx, \tag{7.1}$$

with weight β assigned to the reputational payoff (2.2) and weight $1-\beta$ to the expected quadratic loss resulting from deviations of the action taken from the optimal action. The statistical payoff has the same specification as in Gul and Lundholm (1995).

Clearly, for $\beta = 0$ the forecaster wishes to make the best statistical prediction, so that truthtelling results. In this pure statistical model

$$\frac{\partial}{\partial m} E\left[(E[x|m]-x)^2\,|s\right] = 2\left(E[x|m]-E[x|s]\right)\frac{\partial E[x|m]}{\partial m}. \tag{7.2}$$

Once $m = s$ is substituted in (7.2), truthtelling is verified to be an equilibrium. When instead there is positive weight on the reputational payoff $\beta > 0$, truthtelling cannot be an equilibrium. This follows immediately from our results in Section 3, as the derivative of (7.1) is β times the one found in the pure reputational model.

It is simple to verify that $\partial^2 (E[x|m]-x)^2 / \partial E[s|m]\partial x < 0$ in our multiplicative linear model. This and (5.3) imply that the mixed model (7.1) satisfies the sorting condition.

to our companion paper Ottaviani and Sørensen (2001b). There we develop methods to compare the predictions of the reputational cheap talk theory to those of the forecasting contest theory.

The marginal payoff resulting from a message with higher mean increases in the level of the realized state. We conclude that the partition structure of the equilibrium is robust to the introduction of statistical payoff. We conjecture that the relative importance of the statistical objective determines how fine an equilibrium partition can be.

7.2. Interim Model

In our baseline model we have assumed that the evaluator observes the state of the world. This section considers the case where the evaluator only observes the message sent by the expert, but does not have access to any additional information on the state of the world. Still denoting the evaluator's conjecture of the expert's mixed strategy by $\hat{\mu}(m|s)$, we have $\hat{f}(m|t) = \int_S \hat{\mu}(m|s) f(s|t)\, ds = (1 + E[s|m]tEx)\,\hat{\mu}(m)/2$, so that $p\,(t|m) = \hat{f}(m|t)p(t)/\hat{f}(m) = (1 + E[s|m]tEx)\,p(t)/(1 + E[s|m]EtEx)$. The interim (I) pure reputational payoff from sending m is

$$V^I(m) = \int_T v(t)p(t|m)\, dt = E\left[v\left(t\right)\right] + \left(E\left[tv\left(t\right)\right] - E\left[v\left(t\right)\right]Et\right)\frac{E[s|m]Ex}{1 + E[s|m]EtEx}. \quad (7.3)$$

In this interim reputation model, the only equilibrium is pooling. The indifference condition implies that different messages cannot reveal different information about t. In the linear (as well as in the binary) model, no information about t implies no information about x, so that:

Proposition 11 (Interim Reputation). *In the interim reputation model where $Ex \neq 0$ there is no informative equilibrium, even allowing for mixed strategies.*

Exactly like in the partisan model of Crawford and Sobel (1982), in the interim reputation model the receiver's evaluation action is based exclusively on the message reported by the sender. Some sorting is necessary in signaling games for messages to be credible. For any information at all to be possibly communicated in equilibrium, the evaluator must receive some information about the state in addition to the message sent by the adviser.

This model also relates to Brandenburger and Polak's (1996) analysis of investment decisions by privately informed managers who are concerned with current share price. The current share price in turn reflects the information inferred by the stock market from the manager's observable investment behavior. Our interim model can be seen as a continuous-signal reputational-objective analogue of their model. In their binary-signal model there is no pure-strategy informative equilibrium other than for a degenerate prior on the state (their Proposition 1), but there is an informative mixed-strategy equilibrium for a set of non-degenerate priors on the state (Proposition 2). Their mixed strategy equilibrium has the property that all messages are equally attractive to the sender. Yet, their messages convey some information about the state of the world, something impossible in our reputational context.

We can now revisit the model of Prendergast and Stole (1996), whose prediction of full separation contrasts with ours. Their full revelation result depends on three crucial assumptions: interim payoffs, mixed objective, and delegation. In their model, the manager (expert) cares both about the reputation about ability and the payoff attained with the decision taken by herself. They find that the equilibrium is fully separating and that the decision taken is distorted because of the reputational motive. To illustrate this, consider the mixed interim model with delegation (ID), where

$$V^{ID}(m) = \beta V^{I}(m) - (1-\beta)(m - E[x|s])^2 .$$

Unlike in Section 7.1 evaluation occurs before x is realized. The manager assigns weight β to the interim reputational payoff (7.3) which follows the inference made by the market on the basis of the action taken. Weight $1-\beta$ is assigned to the quadratic loss due to the deviation from the optimal action conditional on all the information available.

We look for a fully separating equilibrium, where the strictly increasing strategy $m(s)$ is differentiable. The necessary first order condition is the differential equation $k = (m(s) - E[x|s])(1 + E[t]E[x]s)^2 m'(s)$, where $k = \beta E[x](E[tv(t)] - E[v(t)]E[t])/2(1-\beta)$. It is easily verified that

$$m(s) = E[x|s] + \frac{\beta E[x](E[tv(t)] - E[v(t)]E[t])}{2(1-\beta)E[t](E[x^2] - E[x]^2)} \tag{7.4}$$

solves the differential equation. Then, $m'(s) > 0$ as assumed, and the second-order condition holds. The term $E[x|s]$ is the expert's honest prediction of the state of the world, and the difference between this and $m(s)$ is a constant bias term. This bias is naturally stronger the more the expert cares about his reputation, and the farther is $E[x]$ from 0. Notice the similarity with the equilibrium characterized by Prendergast and Stole in their normal learning model where the expert knows her own ability.

This fully revealing equilibrium would not survive in a cheap-talk framework where the action taken by the receiver incorporates all the information revealed by the sender. The ex-post optimal decision for the receiver would not be compatible with signal-to-signal incentive compatibility. Interim payoffs with cheap talk are

$$V^{IC}(m) = \beta V^{I}(m) - (1-\beta)(E[x|m] - E[x|s])^2 .$$

We look for a fully separating equilibrium as above. Since the strategy is invertible (i.e. fully separating), the prediction cannot be biased, $E[x|m] = E[x|s]$. Observe then that

$$\frac{\partial}{\partial m}(1-\beta)\left(E[x|m]-E[x|s]\right)^2 = 2\left(1-\beta\right)\left(E[x|m]-E[x|s]\right)\frac{\partial E[x|m]}{\partial m} = 0.$$

When the prediction is unbiased (or the action taken ex-post optimal), the first-order effect on the statistical payoff from changing m is zero. On the other hand, given a truthtelling strategy it is easy to see that $\partial V^I/\partial m$ is non-zero. Thus the equilibrium cannot be fully separating. Partially informative equilibria can be easily constructed.

To summarize, coarseness results in the pure reputational model and truthtelling in the pure statistical model. The predictions with pure objectives are identical, regardless of whether delegation or cheap talk is considered. With mixed objectives, cheap talk results at best in a partially revealing equilibrium (bunching) both in the interim and the ex post case. Full separation with distortion results instead in the mixed interim delegation model.

7.3. Known Own Ability

In our basic model, the expert receives a signal s which is more informative about ability than the message m submitted in equilibrium. In non-trivial dynamic extensions, there would therefore be asymmetric information on ability between the sender and the receivers, as also argued by Avery and Chevalier (1999). In order to study the robustness of our results to the addition of private information on own ability, we now investigate the case in which the expert knows perfectly her own ability type, as first done by Trueman (1994). By adapting Trueman's analysis, Lemma 4 in Ottaviani and Sørensen (2001a) shows that in a binary model there exists always an informative equilibrium, which often involves some randomization by the least able of the two types.

When the expert privately knows not only the signal realization s but also her own ability type t, there cannot be a fully revealing equilibrium whereby both s and t are communicated truthfully. Otherwise, each expert would want to claim to have the highest ability. In the dichotomous model (3.1), the posterior on x is

$$q(x|s,t) = \frac{f(s|x,t)}{f(s|t)}q(x) = \frac{tq(s)}{tq(s)+(1-t)h(s)}\delta_x(s) + \frac{(1-t)h(s)}{tq(s)+(1-t)h(s)}q(x),$$

so that the proof of Proposition 2 can be adapted to show that there is an equilibrium in which the signal is communicated truthfully, $m(s,t) = s$. More generally, truthful reporting of the signal is incompatible with equilibrium as shown in Section 3 for the unknown own ability case.

Consider next the multiplicative linear model (5.1). An expert of ability t who receives signal s has posterior on the state

$$q\left(x|s,t\right)=\frac{f\left(s|x,t\right)}{f\left(s|t\right)}q\left(x\right)=\frac{1+stx}{1+stEx}q(x). \tag{7.5}$$

Denote the conjectured strategy of the sender by $\hat{\mu}(m|s,t)$. Let $\hat{f}(m|x,t)=\int_S \hat{\mu}(m|s,t)\hat{f}(s|x,t)\,ds$. Upon observation of message m and state x, the posterior reputation is $p(t|m,x) = \hat{f}(m|x,t)p(t)/\hat{f}(m|x)$. The expected reputational payoff of message m for a sender with signal s and ability t is

$$V(m|s,t)\equiv\int_X\int_T v(t')p(t'|m,x)\,dt'q\left(x|s,t\right)\,dx. \tag{7.6}$$

Notice that we are assuming that value functions are not ability dependent. This is a strong assumption, because an expert with private information on her own ability knows better than the market how her reputation will be updated in later periods. In a full dynamic model an expert's prospects of future earnings would then depend on ability.

The problem is one of multi-dimensional signaling. Notice from (7.5) that all signal-ability type combinations with $st=k$ constant result in the same posterior belief on the state

$$q\left(x|s,t=k/s\right)=\frac{1+kx}{1+kEx}q\left(x\right). \tag{7.7}$$

The rectangular hyperbola $t=k/s$ represents such iso-posterior locus in the space $S\times T=[-1,1]\times[0,1]$. See Figure 4 for a map of such iso-posterior curves. Truthful reporting of iso-posterior curves is impossible, since the curve $st=1$ implies that $t=1$, and it would yield the perfect posterior reputation.

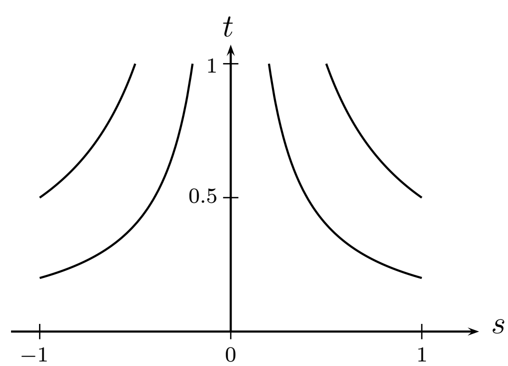

Figure 4: Iso-posterior curves with $k=-1/2$, $k=-1/5$, $k=1/5$, and $k=1/2$.

We focus on equilibria where the threshold of indifference between a message and another are iso-posterior curves. In a binary equilibrium message m is sent for $-1/t\leq$

$s \le k/t$ and message m' is sent for $k/t \le s \le 1/t$. In contrast to the case when the expert does not know her ability, there is an informative equilibrium for *any* prior belief on the state.

Proposition 12 (Known Own Ability). *When the expert knows her own ability, there exists always a binary informative equilibrium.*

Why is there necessarily an informative equilibrium when the expert knows her own type? If some message were sent exclusively by the highest possible type, then it would give the highest possible reputation, regardless of the realized state of the world. More able experts are more confident in their prediction of the state of the world. As the prior belief on the state becomes very skewed, only the strongest types have the self-confidence required to send a message opposite to the prior. If a sufficiently small set of good types are sending the message, they signal that they are good and thereby secure a good minimum reputation, even when the state of the world turns out against them.[27]

7.4. Multiple Experts and Relative Reputational Concerns

> "a decision was made at the outset of this study not to disclose the sources of the forecasts evaluated (...) forecasters are rivals or competitors (...) Any statement bearing on the relative quality of a forecaster's product could be used in this competition." (page 1 in Zarnowitz (1967)).

Can competition between experts affect the amount of information credibly communicated? For instance, full information revelation results in equilibrium when consulting simultaneously multiple perfectly informed experts in the partisan cheap-talk model of Crawford and Sobel (1982), as shown by Krishna and Morgan (2000) and Battaglini (2002). Consider instead multiple professional experts with conditionally independent signals. If they simultaneously report their messages and care only about their own (*absolute*) reputation, the equilibrium is the same as in the single-expert model.[28]

In our reputational setting it is quite natural to allow for *relative* performance evaluation. Often, the market rewards those with better reputation more if they are scarcer. One could expect that in our setting more differentiation (and perhaps more information revelation in our model with privately information experts) would result when a concern

[27] In a simple example (with $v(t) = t$ and $p(t)$ uniform on $[0, 1]$) it can be shown that there is only one equilibrium within the binary class, in contrast with the typical multiplicity of binary equilibria found in the case of unknown ability.

[28] See also Levy (2000) and Ottaviani and Sørensen (2001a) on issues arising when consulting sequentially professional experts with absolute reputational concerns.

for relative reputation is introduced. We now show that this is not the case when reputational preferences have a von Neumann-Morgenstern representation and experts have conditionally independent signals.

Consider this model where experts $i = 1, ..., N$ report simultaneously. With relative reputational concerns, the von Neumann-Morgenstern payoff $u^i(t^i, t^{-i})$ to expert i depends also on the ability of all other experts $t^{-i} \equiv \left(t^1, \ldots, t^{i-1}, t^{i+1}, \ldots, t^N\right)$. We maintain the assumption that u^i is increasing in t^i, but it might well be decreasing in t^j for $j \neq i$. We make the natural assumption that the state of the world and the ability types of the experts are independently distributed. As customary in information economics, we further assume that the noisy signals of experts are *independent* conditionally on the state and ability draws.[29]

The independence assumptions implies stochastic independence of posterior reputations of different experts updated after the reports and observation of the state of the world. Moreover, only an expert's own message (and the state of the world) influences the updating of the reputation of that expert. According to the martingale property of updated Bayesian beliefs, the expected posterior reputations of other experts equal the prior reputations. Finally, the von Neumann-Morgenstern payoff is linear in those beliefs. Thus we have the next general result:

Theorem 2 (Irrelevance of Relative Reputation). *Assume that the experts have von Neumann-Morgenstern payoffs, and that their signals are independent conditionally on state and ability. In equilibrium of the relative reputation model, expert i behaves as in the absolute reputation model with increasing value function*

$$v^i(t^i) = E_{t^{-i}}[u^i(t^i, t^{-i})]$$

Notice that this result does not rest on our functional assumptions about $f(s|x, t)$ and holds even if each expert privately knows her own ability. According to this theorem, in order to generate new and interesting results a relative reputations model must either assume that there is correlation of experts' signals conditionally on the state and ability draw or give up the von Neumann-Morgenstern formulation. For an investigation of relative reputational concerns in a binary model with conditionally correlated signals see Effinger and Polborn (2001). Notice that the von Neumann-Morgenstern formulation is

[29]Notice that in this way we depart from the route taken by a large part of the reputational herding literature that assumed better managers to have more correlated signals conditional on the state of the world since Scharfstein and Stein (1990).

rather restrictive in this setting. Naturally, the market might instead reward experts on the basis of a comparison of some summary statistics of their updated reputation.[30]

In the linear model we can conclude that the equilibrium behavior of each expert is unaffected by the introduction of relative reputation evaluation. This follows from the special property of the linear model that the equilibrium is invariant to monotonic transformations of the value function, as seen in equation (4.2).[31]

Proposition 13 (Equilibrium Equivalence with Relative Reputation). *In the multiplicative linear model with conditional independence across experts, the equilibrium in the relative reputation model is the same as in the absolute reputation model.*

8. Conclusion

Increasing specialization of labor suggests that information should be collected by professional experts. This paper studies information transmission by a partially-informed expert who wishes to be perceived as well informed. The evaluator cannot commit but to use ex post all the available information to assess the expert's ability and is not allowed to design contingent monetary rewards. Compared to the partisan expert case, the analysis is complicated by the necessary presence of simultaneous learning on state and ability. We have managed to make the problem tractable by imposing more restrictive assumptions as the analysis proceeded and focusing on special but natural cases.

Our first result is that truthtelling/full separation is generally not an equilibrium when the signal and the state can be cross checked to update beliefs about the expert's type. In a putative fully revealing equilibrium, the signal and the realized state are informative about the expert's type, giving an incentive to the expert to mis-report the signal in order to generate a better reputation. In contrast with the canonical model of partisan advice, truthtelling in a professional setting is possible, but only under non generic conditions. More precisely, we have shown that if there is some interval of signals that is truthfully reported in equilibrium for an open and dense set of priors over the state and value functions, then the signal structure must satisfy the very special property of local uninformativeness.

Second, in order to improve our understanding of a professional expert's incentive to manipulate the market beliefs we considered the optimal reporting strategy for the expert if the evaluator wrongly believes that the expert is telling the truth. We have shown that

[30] For instance, the case in which the expert with highest expected ability receives all the rewards cannot be modeled with von Neumann-Morgenstern payoffs.

[31] This proposition applies only to the case of initial symmetric information about the expert's ability. When the expert has private information about ability, the equilibrium depends instead on the value function.

in an important special case, the expert has an incentive to deviate toward the neutral signal. Intuitively, a more conservative signal is more likely to be close to the realized state and so results in a more favorable update on ability. When the market evaluates performance in this naive way, excessive conformism results. This is clearly incompatible with equilibrium behavior.

Third, the recommendations provided in equilibrium by experts motivated by their reputation as good forecasters are less accurate than the information they possess. The exact amount of information that can be credibly conveyed depends on the nature of the information possessed by the expert. By looking at the case in which the expert receives a binary signal of varying intensity, we have shown that equilibria can be taken to have a partition structure, the most informative equilibria are typically multiple and they involve no more than two messages. When the prior is sufficiently concentrated on any state, there is only a pooling equilibrium. These striking results could not be easily foreseen in a two-signal model, but hold in this natural generalization.

Finally, we have extended the model in a number of ways to investigate how the results generalize to more realistic environments. We have shown that (i) communication is also coarse when the expert has a partial direct concern for forecast accuracy, (ii) an expert exclusively concerned with reputation cannot communicate any information if the evaluator does not observe the state, and (iii) it is always possible to sustain a binary informative equilibrium when the expert privately knows her own ability type. We have also shown that relative reputational concerns are irrelevant if the payoff has a von Neumann-Morgenstern specification and the signals are conditionally independent.

We conclude that while the implicit incentives provided by the market discipline the expert's behavior, they also increase the scope for strategic manipulation in the revelation of a given level of information. It is natural to ask how these problems can be overcome with optimally designed explicit incentives. A starting point for investigating the interaction between explicit and implicit incentives is provided by Holmström and Ricart i Costa (1986) for the case in which ability adds instead to the value of the output produced. It would also be interesting to extend our model to allow the expert to become more informed by acquiring costly signals. The point of departure would be Osband's (1989) study of explicit incentives for truthtelling and information acquisition by forecasters in the absence of reputational concerns.

Appendix A: Proofs

Proof of Proposition 1 (Truthtelling in Location Experiment). Since the receiver anticipates truthtelling, the posterior reputation is $p(t|m,x) = p(t)\, f(m|x,t) / f(m|x) = p(t)\, g(m-x|t) / g(m-x)$. By the location property $p(t|m,x)$ depends only on the difference $m-x$, and by symmetry it depends only on $|m-x|$. By the MLRP, the smaller is $|m-x|$ the better news for t, i.e. $p(t|m,x)$ is better in the first-order stochastic dominance sense. For any increasing v we conclude that $W(m|x) \equiv \int_T v(t)p(t|m,x)\,dt$ depends only on $|m-x|$, and is a decreasing function of $|m-x| \in [0,\pi]$.

Since $q(x)$ is the uniform distribution, the sender's posterior belief on x is described by the p.d.f. $q(x|s) = f(s|x)/f(s)$. By symmetry and unimodality of $f(s|x)$, this distribution of x is symmetric and unimodal around s. Thus, $q(x|s)$ depends only on $|x-s|$ and is decreasing in $|x-s| \in [0,\pi]$.

We now show that these properties of W and q imply that $m = s$ maximizes $V(m|s) = \int_X W(m|x)q(x|s)\,dx$ over m, so that truthtelling is optimal. By symmetry, it suffices to consider $m \in [s, s+\pi]$ and prove that $V(s|s) \geq V(m|s)$. Note that half of the space X is closer to s than to m, namely the values of x in the interval $[(s+m-2\pi)/2, (s+m)/2]$. We have

$$
\begin{aligned}
V(s|s) - V(m|s) &= \int_{(s+m-2\pi)/2}^{(s+m)/2} \left[W(s|x) - W(m|x)\right] q(x|s)\ dx \\
&\quad + \int_{(s+m)/2}^{(s+m+2\pi)/2} \left[W(s|x) - W(m|x)\right] q(x|s)\ dx \\
&= \int_{(s+m-2\pi)/2}^{(s+m)/2} \left[W(s|x) - W(m|x)\right] q(x|s)\ dx \\
&\quad + \int_{(s+m-2\pi)/2}^{(s+m)/2} \left[W(s|s+m-x) - W(m|s+m-x)\right] q(s+m-x|s)\ dx \\
&= \int_{(s+m-2\pi)/2}^{(s+m)/2} \left[W(s|x) - W(m|x)\right] \left[q(x|s) - q(x|m)\right]\ dx
\end{aligned}
$$

where the first equality is by definition, the second uses the change of variable $y = m+s-x$ in the second integral, and the last follows from W and q depending on their arguments only through their distance. Since $[(s+m-2\pi)/2, (s+m)/2]$ is the interval of x values closer to s than m, we have $W(s|x) \geq W(m|x)$ and $q(x|s) \geq q(x|m)$. Then the integrand is always non-negative and so the integral is non-negative, proving $V(s|s) - V(m|s) \geq 0.\square$

Proof of Proposition 2 (Truthtelling in Dichotomous Experiment). It is seen immediately that $f(s|x) = (Et)\delta_x(s) + (1-Et)h(s)$ and $f(s) = (Et)q(s) + (1-Et)h(s)$.

Assuming truthtelling, the posterior reputation is

$$p(t|m,x)=\frac{f(m|x,t)}{f(m|x)}p(t)=\begin{cases}\frac{tp(t)}{Et} \text{ if } x=m\\ \frac{(1-t)p(t)}{(1-Et)} \text{ if } x\neq m\end{cases}$$

The reputation obtained after the realization $x=m$ dominates in the first-order stochastic sense the reputation following $x\neq m$. The posterior on x is

$$q(x|s)=\frac{f(s|x)}{f(s)}q(x)=\frac{(Et)q(s)}{(Et)q(s)+(1-Et)h(s)}\delta_x(s)+\frac{(1-Et)h(s)}{(Et)q(s)+(1-Et)h(s)}q(x),$$

an average between an atom at $x=s$ and the continuous prior $q(x)$.

By sending $m=s$ there is a positive probability that $x=m$ and the reputation will be favorably updated. If instead $m\neq s$ there is probability zero that $x=m$, so that the updating which results is necessarily unfavorable. The sender prefers the chance of a favorable updating and so truthfully reports $m=s$. □

Proof of Theorem 1 (No Truthtelling). The set for which truthtelling holds in an interval is defined by the collection of weak inequalities $V(s|s)\geq V(m|s)$ for all $m\in S, s\in I$. By continuity of the integrals w.r.t. the prior belief $q(.)$ and value function $v(.)$, this set is closed. The complement of this set is the set of q,v for which local truthtelling is impossible and so it is open.

The set is also dense, since from any pair q,v it is possible to find another pair q',v' arbitrarily close to q,v such that the local truthtelling fails. This is shown analytically by differentiating (3.2). We have

$$V_m(m|s)=\int_X W_m(m|x)q(x|s)\,dx=\int_X\left[\int_T v(t)p_m(t|m,x)\,dt\right]q(x|s)\,dx,$$

where

$$p_m(t|m,x)=p(t)\,\frac{f_s(m|x,t)\,f(m|x)-f(m|x,t)\,f_s(m|x)}{[f(m|x)]^2}.$$

so that

$$V_m(s|s)=\int_X\int_T v(t)p(t)\,\frac{f_s(s|x,t)\,f(s|x)-f(s|x,t)\,f_s(s|x)}{f(s|x)\,f(s)}\,dtq(x)\,dx.$$

The identity $V_m(s|s)=0$ can be rewritten as

$$0=\int_X\int_T v(t)p(t)\,\frac{f_s(s|x,t)\,f(s|x)-f(s|x,t)\,f_s(s|x)}{f(s|x)}\,dt\,q(x)\,dx.$$

If truthtelling holds for all $s\in I$ under local perturbations in q, then for almost all $x\in X$:

$$0=\int_T v(t)p\left(t\right)\frac{f_s\left(s|x,t\right)f\left(s|x\right)-f\left(s|x,t\right)f_s\left(s|x\right)}{f\left(s|x\right)}\,dt.$$

If truthtelling is further robust against local perturbations in v, then for all $s\in I$ and almost all $x\in X$ and almost all $t\in T$:

$$0=p(t)\frac{f_s\left(s|x,t\right)f\left(s|x\right)-f\left(s|x,t\right)f_s\left(s|x\right)}{f\left(s|x\right)}$$

This implies for all $s\in I$ and almost all $x\in X$ and almost all $t\in T$:

$$\frac{f_s\left(s|x,t\right)}{f\left(s|x,t\right)}=\frac{f_s\left(s|x\right)}{f\left(s|x\right)}.$$

This condition states that the ratio $f_s\left(s|x,t\right)/f\left(s|x,t\right)$ does not depend on t. The ratio $f_s\left(s|x,t\right)/f\left(s|x,t\right)$ is equal to $d\log\left(f\left(s|x,t\right)\right)/ds$, so through integration it determines $\log\left(f\left(s|x,t\right)\right)$ up to an additive constant. Thus we can conclude that there exist functions $K\left(t\right)$ and $g\left(s|x\right)$ such that at all $s\in I$ and almost all $x\in X$ and almost all $t\in T$ we have

$$f\left(s|x,t\right)=K\left(t\right)g\left(s|x\right).$$

The signal is then locally uninformative, in violation of the assumption. □

Proof of Proposition 3 (Best Deviation). Observe that any sender who reports truthfully has expected value Ev:

$$V(s|s)=\int_T v(t)\int_X p(t|s,x)q(x|s)\,dx\,dt=\int_T v(t)p(t|s)\,dt=Ev(t).$$

Now, fix $s>\tilde{s}$ without loss of generality. We argue that the sender with s can profitably deviate to any signal $s'\in(\tilde{s},s)$. Reporting s' gives the expected reputational value

$$V(s'|s)=\int_X\int_T v(t)p(t|s',x)\,dt\,q(x|s)\,dx$$

to be compared with the truthtelling value $V(s|s)$. We will argue that $V(s'|s)>V(s|s)$ for any $s'\in(\tilde{s},s)$, as this proves the incentive to deviate from s to s'. Since $V(s'|s')=V(s|s)$, we can equivalently show $V(s'|s)>V(s'|s')$.

Our comparison rests on two facts. First, since $s>s'$, $q(x|s)$ first-order stochastically dominates $q(x|s')$. Second, with the signal $s'>\tilde{s}$, the higher is the state of the world, the more favorable the updated reputation, so that

$$W\left(s'|x\right)=\int_T v(t)p(t|s',x)\,dt$$

is an increasing function of x. This follows from Milgrom's (1981) Proposition 1 because: x is good news for t when $s' > \tilde{s}$,

$$\frac{p(t'|s',x')}{p(t|s',x')} = \frac{f(s'|x',t')}{f(s'|x',t)}\frac{p(t')}{p(t)} > \frac{f(s'|x,t')}{f(s'|x,t)}\frac{p(t')}{p(t)} = \frac{p(t'|s',x)}{p(t|s',x)}$$

for $t' > t$ and $x' > x$, as a consequence of Lemma 2 proved below; and $v(t)$ is increasing. Combining the two facts we reach the desired

$$V(s'|s) = \int_X \int_T v(t)p(t|s',x)\,dtq(x|s)\,dx > \int_X \int_T v(t)p(t|s',x)\,dtq(x|s')\,dx = V(s'|s').$$

Finally, we show that deviating to any s' outside the interval $(\tilde{s}, s)$ is not profitable. First, for $s' \geq s$, the first fact is reversed, since s' is better news than s for x. This in turn reverses the final inequality, making the deviation unattractive. Second, consider $s' \leq \tilde{s}$. The second fact above is reversed, as higher x is worse news about ability when $s' \leq \tilde{s}$. We can conclude that the best deviation is to some $s' \in (\tilde{s}, s)$. $\square$

In the previous proof we have used the following result:

Lemma 2. *Consider the linear model with $g_{sx} > 0$ and neutral signal $\tilde{s}$. Let $t' > t$ and $x' > x$. Then*

$$\frac{f(s|x',t')}{f(s|x',t)} > \frac{f(s|x,t')}{f(s|x,t)} \tag{A.1}$$

for all $s > \tilde{s}$.

Proof. Substituting $f(s|x,t) = tg(s|x) + (1-t)h(s)$, (A.1) is equivalent to

$$\frac{t'g(s|x') + (1-t')h(s)}{tg(s|x') + (1-t)h(s)} > \frac{t'g(s|x) + (1-t')h(s)}{tg(s|x) + (1-t)h(s)}$$

or

$$(t'-t)\left[g(s|x') - g(s|x)\right] > 0, \tag{A.2}$$

for $t' > t$, $x' > x$, and $s > \tilde{s}$. Notice that $g_x(\tilde{s}|x) = 0$ for all x and $g_{sx} > 0$ imply that $g_x(s|x) > 0$ for $s > \tilde{s}$, so that (A.2) holds. $\square$

Proof of Proposition 4 (Absolutely No Truthtelling). When the strategy is truthtelling, $E[s|m] = m$. Using (5.2), the first order condition for truthtelling $0 = V_m(s|s) = \int_X W_s(s|x)\,f(s|x)q(x)\,dx$ reduces to

$$\int_X \frac{x}{(1+sxEt)}q(x)\,dx = 0.$$

Assume that this equation were to hold at two signals, $s' > s$. Subtracting the two equations yields the contradiction

$$0 = \int_X \frac{(s' - s)x^2}{(1 + sxEt)(1 + s'xEt)} q(x)\, dx > 0,$$

where the strict inequality follows from the integrand being positive for all x. $\square$

Proof of Proposition 5 (Monotonicity). Notice that $f(s|x)$ satisfies the MLRP and $W(m'|x) - W(m|x)$ is increasing in x. It then follows immediately from Proposition 1 of Milgrom (1981) that

$$V(m'|s) - V(m|s) = \int_X [W(m'|x) - W(m|x)] q(x|s)\, dx$$

increases in s. $\square$

Proof of Proposition 7 (Partition Size). Suppose that three or more distinct messages were sent in equilibrium. Proposition 1 shows that there must be at least two message intervals with a non-empty interior, except if the equilibrium has message intervals $[-1, -1], (-1, 1), [1, 1]$. In this very case, there is probability one of observing message $(-1, 1)$, so this is equivalent to a one-message equilibrium. For the remainder of the proof we can assume that three signal intervals $[a, b]$, $[b, c]$, $[c, d]$ define equilibrium messages, with $a < b \leq c < d$.

By incentive compatibility and payoff continuity, an individual with signal $s = b$ must be indifferent between the $[a, b]$ and $[b, c]$ messages, i.e. $V([a, b]|b) = V([b, c]|b)$. Using (4.2), in the multiplicative linear model this condition can be rewritten as

$$\int_X \frac{\hat{\mu}\left([a, b]\right) \hat{\mu}\left([b, c]\right) x f(b|x)}{\hat{f}([a, b]|x)\hat{f}([b, c]|x)} q\left(x\right)\, dx = 0. \tag{A.3}$$

Indifference between messages $[b, c]$ and $[c, d]$ at signal c gives an analogous condition, which subtracted from (A.3) gives

$$\int_X \left[\frac{\hat{\mu}\left([a, b]\right) f(b|x)}{\hat{f}([a, b]|x)} - \frac{\hat{\mu}\left([c, d]\right) f(c|x)}{\hat{f}([c, d]|x)} \right] \frac{\hat{\mu}\left([b, c]\right) x}{\hat{f}([b, c]|x)} q(x)\, dx = 0. \tag{A.4}$$

The integrand factor $\hat{\mu}\left([b, c]\right) x / \hat{f}([b, c]|x)$ vanishes at $x = 0$, the neutral state of the world where signals are non-informative about type. When x is positive, the term is positive, and vice versa when x is negative.

The other integrand factor in (A.4), $\hat{\mu}([a,b])\, f(b|x)/\hat{f}([a,b]|x) - \hat{\mu}([c,d])\, f(c|x)/\hat{f}([c,d]|x)$ also vanishes at $x = 0$ since all signals are equally likely. The MLRP integrates to prove that it is better news about x to observe signal b than to observe that the signal was in the range $[a,b]$. This MLRP means that $f(b|x)/\hat{f}([a,b]|x)$ is increasing in x. Likewise, signal c is worse news for x than the interval $[c,d]$, and $f(c|x)/\hat{f}([c,d]|x)$ is decreasing in x.

Since both factors in the integrand have the same sign as x, the integrand and therefore the integral in (A.4) is positive, providing a contradiction. □

Proof of Proposition 9 (No Informative Equilibrium). Define

$$\varphi(a, Et, x) = \frac{x(1 + axEt)}{[2 + (a-1)xEt]\,[2 + (a+1)xEt]} \tag{A.5}$$

as the integrand of (5.4). Consider an arbitrary $\tilde{x} \in (0,1]$. Then for all $a \in (-1,1)$, $Et \in [0,1]$, we have $\varphi(a, Et, \tilde{x}) \geq \tilde{x}/8 > 0$ since $2(1 + a\tilde{x}Et) \geq 2 + (a-1)\tilde{x}Et > 0$, and $4 \geq 2 + (a+1)\tilde{x}Et > 0$. By continuity, for x close enough to $\tilde{x}$, $\varphi(a, Et, x)$ is positive and bounded away from zero. When the prior on the state is sufficiently concentrated on $\tilde{x}$, there is no solution to the indifference equation (5.4), since the left-hand side is positive for any a. An analogous argument applies to negative states. □

Proof of Proposition 10 (Comparative Statics). First, the left-hand side of (5.4) decreases with a near $a = 0$ since its derivative in a evaluated at $a = 0$ is

$$\int_X \frac{-x^4(Et)^3 - 2x^3(Et)^2}{[4 - x^2(Et)^2]^2} q(x)\, dx.$$

Evaluated at the original symmetric distribution of x, this integral is negative, since $l(x) = x^3/\left(4 - x^2(Et)^2\right)^2$ has the antisymmetry property $l(-x) = -l(x)$.

Second, integrand $\varphi(a, Et, x)$ of (5.4) defined in (A.5) is increasing in x near $a = 0$, since its derivative with respect to x computed at $a = 0$ is $(4 + x^2(Et)^2)\,/\,(4 - x^2(Et)^2) > 0$. By Milgrom's (1981) Proposition 1, a FSD increase in the distribution of x will raise the LHS of (5.4). By the implicit function theorem, locally there will still be an equilibrium, and the equilibrating threshold a must increase with this FSD upward shift. When the prior state-belief is changed in favor of higher states of the world, some experts change from sending the high message m' to sending the low message m. Furthermore, since the derivative of the LHS in a is negative, and since Ex changes to become positive, there must appear a second binary equilibrium with a threshold closer to 1. □

Proof of Proposition 11 (Interim Reputation). Any two messages m and m' sent with positive probability in equilibrium must give the same payoff, as otherwise the message with lowest payoff would not be sent. The condition $V(m) = V(m')$ is equivalent to $E[s|m]Ex = E[s|m']Ex$, and since $Ex \neq 0$ the messages satisfy $E[s|m] = E[s|m']$. Now all messages sent in equilibrium have the same average signal, and this common average must be zero, since 0 is the overall average signal. Then no message conveys any information about t or x. $\square$

Proof of Proposition 12 (Known Own Ability). We establish this result through two Lemmas.

Lemma 3. *There exists a $k \in (-1, 1)$ such that $V(m|s, t = k/s) = V(m'|s, t = k/s)$ when m is sent by $st \leq k$ types and m' is sent by the others.*

Proof. First, we argue that for k sufficiently close to 1, the sender with $st = k$ prefers message m' over m. As k tends to one, the set m' of (s, t) satisfying $st \geq k$ shrinks towards the corner $(s, t) = (1, 1)$. So, $\hat{\mu}(m')$ tends to zero, while $\hat{\mu}(m)$ tends to one. Since m tends towards an uninformative message, we have $W(m|x)$ tending to $Ev(t)$ for all x. For any $\varepsilon > 0$ there exists a $k^* \in (0, 1)$ such that $V(m|s, t) < Ev(t) + \varepsilon$ for all $k > k^*$. Since v is increasing and the prior distribution of t is not degenerate, there exists some $t^* \in (0, 1)$ with $v(t^*) > Ev(t)$. When $k > t^*$, message m' is sent only by types who know $t > t^*$, so that $V(m'|s, t) > v(t^*)$. Let $\varepsilon = (v(t^*) - Ev(t))/2$ and choose the k^* defined above. When $k > \max\langle t^*, k^*\rangle$ then $V(m|s, t) < Ev(t) + \varepsilon < v(t^*) < V(m'|s, t)$. This is true for all pairs (s, t), and in particular for all those with $st = k$.

By analogy, when k is sufficiently close to -1, the sender with $st = k$ prefers message m over m'. When k changes, we continuously change $V(m|s, t = k/s)$ and $V(m'|s, t = k/s)$. We know that $V(m|s, t = k/s) - V(m'|s, t = k/s)$ is positive for k near -1 and negative for k near $+1$. There must be an intermediate equilibrating k. $\square$

For incentive compatibility, the next argument (similar to Proposition 5) proves that for fixed definitions of messages, $V(m|s, t) - V(m'|s, t)$ is monotonic across iso-posterior curves (we have already observed that it is constant on iso-posterior curves).

Lemma 4. *If the receiver believes that m is sent by the $st \leq k$ types and that m' is sent by the others, then $V(m'|s, t = l/s) - V(m|s, t = l/s)$ is increasing in l.*

Proof. We first prove that $P(t|m,x)$ increases in x when $k \geq 0$, but an analogous argument works for $k < 0$. From

$$\hat{f}(m|x,t) = \int_{-1}^{k/t} f(s|x,t)\,ds = \int_{-1}^{k/t} \frac{1+stx}{2}\,ds = \frac{I_1(t) + I_2(t)tx}{2},$$

where

$$I_1(t) = \int_{-1}^{k/t} ds = \begin{cases} 2 & \text{if } 0 \leq t \leq k \\ 1 + \frac{k}{t} & \text{if } k \leq t \leq 1, \end{cases} \tag{A.6}$$

and

$$I_2(t) = \int_{-1}^{k/t} s\,ds = \begin{cases} 0 & \text{if } 0 \leq t \leq k \\ \frac{1}{2}(\frac{k^2}{t^2} - 1) & \text{if } k \leq t \leq 1, \end{cases} \tag{A.7}$$

we have

$$\hat{f}(m|x) = \int_0^1 \hat{f}(m|x,t)p(t)\,dt = \frac{E\left[I_1(t)\right] + E\left[I_2(t)t\right]x}{2}.$$

Then

$$p(t|m,x) = \frac{I_1(t) + I_2(t)tx}{E\left[I_1(t)\right] + E\left[I_2(t)t\right]x}p(t)$$

and

$$P(t|m,x) = \frac{E\left[I_1(t) \mid t' \leq t\right] + E\left[I_2(t)t \mid t' \leq t\right]x}{E\left[I_1(t)\right] + E\left[I_2(t)t\right]x}P(t). \tag{A.8}$$

(A.7) implies that I_2 is non-positive and decreasing in t, so that $0 \geq E\left[I_2(t)t \mid t' \leq t\right] \geq E\left[I_2(t)t\right]$. Similarly, $I_1(t)$ is positive and decreasing in t so that $E\left[I_1(t) \mid t' \leq t\right] \geq E\left[I_1(t)\right] \geq 0$. Then (A.8) shows that $P(t|m,x)$ is increasing in x. It follows that

$$W(m|x) = \int_T v(t)p(t|m',x)\,dt$$

is decreasing in x, as $v(t)$ is increasing in t. Finally, notice from (7.7) that an increase in l yields a first-order stochastic dominance increase in $q(x|s,t=l/s)$, so that in turn this decreases in l:

$$V(m|s,t=l/s) = \int_X W(m|x)q(x|s,t=l/s)\,dx.$$

Similar calculations for message m' show that $P(t|m',x)$ is decreasing in x. Arguing as above, $V(m'|s,t=l/s)$ is then increasing in l. We conclude that $V(m'|s,t=l/s) - V(m|s,t=l/s)$ is increasing in l. $\square$

Proof of Theorem 2 (Irrelevance of Relative Reputation). The reputational value of message m^i for expert i with signal realization s^i is

$$V^i(m^i|s^i) = \int_X W^i\left(m^i|x\right) q(x|s^i)\, dx \tag{A.9}$$

where

$$W^i\left(m^i|x\right) = \int_{M^{-i}} \int_{T^i} \int_{T^{-i}} u^i(t^i, t^{-i}) p(t^{-i}|m, x)\, dt^{-i} p(t^i|m, x)\, dt^i \hat{f}\left(m^{-i}|x\right)\, dm^{-i}.$$

By the assumption of independence of the ability of different experts, posterior reputations are again stochastically independent. In particular, the posterior reputation of an expert is stochastically independent of the message reported by another expert, $p(t^{-i}|m^i, m^{-i}, x) = p(t^{-i}|m^{-i}, x)$ and $p(t^i|m^i, m^{-i}, x) = p(t^i|m^i, x)$, so that

$$\begin{aligned} W^i\left(m^i|x\right) &= \int_{T^i} \int_{M^{-i}} \int_{T^{-i}} u^i(t^i, t^{-i}) p(t^{-i}|m^{-i}, x)\, dt^{-i} \hat{f}\left(m^{-i}|x\right)\, dm^{-i} p(t^i|m^i, x)\, dt^i \\ &= \int_{T^i} v^i\left(t^i, x\right) p(t^i|m^i, x)\, dt^i, \end{aligned}$$

where

$$v^i\left(t^i, x\right) = \int_{T^{-i}} u^i(t^i, t^{-i}) \int_{M^{-i}} p(t^{-i}|m^{-i}, x) \hat{f}\left(m^{-i}|x\right)\, dm^{-i}\, dt^{-i}.$$

The law of iterated expectations gives

$$\int_{M^{-i}} p(t^{-i}|m^{-i}, x) \hat{f}\left(m^{-i}|x\right)\, dm^{-i} = p(t^{-i}|x) = p(t^{-i}),$$

where we also used and the independence of t^{-i} and x. It follows that

$$v^i\left(t^i, x\right) = \int_{T^{-i}} u^i(t^i, t^{-i}) p(t^{-i})\, dt^{-i}.$$

and thus $v^i\left(t^i, x\right) = v^i\left(t^i\right)$ does not depend on x. Furthermore, since u^i is increasing in t^i for any t^{-i}, we find that $v^i(t^i)$ is an increasing function of t^i. We are thus back to the original problem with absolute reputational concerns, with the individual objective function $v^i(t^i) = E_{t^{-i}} u^i(t^i, t^{-i})$.

When expert i knows her own type, equation (A.9) becomes

$$V^i(m^i|s^i, t^i) = \int_X W^i\left(m^i|x\right) q(x|s^i, t^i)\, dx$$

and the rest of the proof goes through as before. $\square$

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